Ch.1 (Relations and Functions)

1. If f: R→R be given by $f(x) = (3-x^3)^{\frac{1}{3}}$, then find (f o f) x.

2. If the function f : R → R be given by $f(x) = x^2 + 2$ and g : R → R be given by $g(x) = \frac{x}{x-1}$, $x \neq 1$, find fog and gof and hence find fog(2) and gof(-3)

3. Let f, g : R → R be defined as f (x) = [x], and g (x) = |x|, find the values of
(i) $(gof)\left(-\frac{5}{3}\right)-(fog)\left(-\frac{5}{3}\right)$ (ii) $(gof)\left(\frac{5}{3}\right)-(fog)\left(\frac{5}{3}\right)$ (iii) (f+2g)(-1)

4. Consider a function f : $R_+ \to [15,\infty)$ given by $f(x) = 4x^2 + 12x + 15$. Show that f is bijective function. Also find f^{-1}.

5. Let f: X→ Y be a function.Define a relation R in X given by R={(a,b):f(a)=f(b)}. Examine whether R is an equivalence relation or not.

6. Let A = {1,2,3,..., 9} and R be the relation in A X A defined by (a, b) ^{R}(c , d) iff
a + d = b + c for all a, b, c ,d ∈ A .Prove that R is an equivalence relation. Also obtain the equivalence class [(2,5)]

7. Let R be a relation on the set A = N×N , where N is the set of natural numbers, defined by (x, y) R (u, v) if and only if x v = y u. Show that R is an equivalence relation.

8. Show that the relation R in the set N×N, defined by (a, b) R (c, d) iff
$a^2 + d^2 = b^2 + c^2 \forall$ a,b,c,d ∈ N, is an equivalence relation.

9. Consider a function f : $R_+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$, prove that that f
is invertible with $f^{-1}(y) = \frac{\sqrt{54+5y}-3}{5}$

10. Let N denote the set of all natural numbers and R be the relation on N×N defined by (a, b) R (c, d) iff ad(b+c) = bc(a+d). Show that R is an equivalence relation.

11. Show that the relation R in the set R of real numbers, defined as
R = {(a, b) : 1 + ab ≥ 0} is reflexive nor symmetric but not transitive.

12. Let f : R→R be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that f is not one-one by giving a counter example.

13. Give the smallest relation on set of Natural numbers which is not reflexive

but symmetric and transitive.

14. Show that composition of two one-one functions is one-one. Is the converse true? Justify.

15. Show that composition of two onto functions is onto. Is the converse true? Justify.

ANSWERS

1. x 2. $6, \frac{11}{10}$ 3. 1,0,1 4. $f^{-1}(y)=\frac{\sqrt{y+6}-3}{2}$ 5. yes

6. [(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)] 13. Ø(Empty set) 14.converse is not true

15. Converse is not true.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

11. Show that : $\tan^{-1}\left[\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos}-\sqrt{1-\cos x}}\right]=\frac{\pi}{4}+\frac{x}{2};\ x\in[0,\pi]$

12. Prove that :

$$\tan^{-1}\left(\frac{\cos x}{1-\sin}\right)-\cot^{-1}\left(\sqrt{\frac{1+\cos}{1-\cos}}\right)=\frac{\pi}{4}\quad x\in(0,\pi/2).$$

13. Prove that $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)=\sin^{-1}\frac{x}{a}=\cos^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)$.

14. prove that:

$$\cot^{-1}\left[2\tan\left(\cos^{-1}\frac{8}{17}\right)\right]+\tan^{-1}\left[2\tan\left(\sin^{-1}\frac{8}{17}\right)\right]=\tan^{-1}\left(\frac{300}{161}\right)$$

15. Prove that:

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2.$$

16. Solve:

$$\cot^{-1}2x+\cot^{-1}3x=\frac{\pi}{4}$$

17. Prove that:

$$\tan\left[\frac{\pi}{4}+\frac{1}{2}\tan^{-1}\left(\frac{a}{b}\right)\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\tan^{-1}\left(\frac{a}{b}\right)\right]=\frac{2\sqrt{a^2+b^2}}{b}$$

18. Solve for x, $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)+\tan^{-1}\left(\frac{-2x}{1-x^2}\right)=\frac{2\pi}{3}$

19. Prove that: $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

20. Solve for x, $\tan(\cos^{-1}x)=\sin(\tan^{-1}2)\,;\ x>0$

21. If $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$, then prove that $\sin y = tan^2\left(\frac{x}{2}\right)$

22. Prove that:

$$\cot\left\{\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)\right\} + \cos^{-1}(1-2x^2) + \cos^{-1}(2x^2-1) = \pi, x > 0$$

23. Prove that:

$$\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \tan^{-1}\left(\frac{c-a}{1+ca}\right) = 0 \text{ where a, b, c} > 0$$

24. If $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$,then

prove that a + b + c = abc

25. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$

[Hint: Let $\cos^{-1}x$ =A, $\cos^{-1}y = B, \cos^{-1}z = C$ then A + B + C = π or A +B = π-c take cos on both the sides].

26. If $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \cdots + \tan^{-1}\left(\frac{1}{1+n.(n+1)}\right) = \tan^{-1}\phi$ then find the value of ϕ.

27. If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$ then find x.

28. If $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1}x)$, then find x.

29. Solve the following for x

(i) $\sin^{-1}(6x) + \sin^{-1}(6\sqrt{3}\,x) = -\frac{\pi}{2}$

(ii) $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

(iii) $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$

(iv) $\sin^{-1}\left(\frac{x}{2}\right) + \cos^{-1}x = \frac{\pi}{6}$

30. If $\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\alpha$,then prove that

$9x^2$-12xycos $\propto +4y^2 = 36\sin^2\propto$

31. Prove that: $\tan^{-1}\left[\frac{3\sin 2\emptyset}{5+3\cos 2\emptyset}\right]+\tan^{-1}\left[\frac{1}{4}\tan\emptyset\right]=\emptyset$

32. Prove that: $\cot^{-1}\left[\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\frac{1}{\sqrt{2}}\right)\right]=\frac{\pi}{2}$

33. Prove that: $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)=\cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$

34. Prove that: $2\tan^{-1}[\tan\propto/2\ \tan\beta/2]=\cos^{-1}\left[\frac{\cos\propto+\cos\beta}{1+\cos\alpha\cos}\right]$

ANSWERS

1. (i) $-\frac{\pi}{3}$ (ii) $\frac{\pi}{6}$ (iii) $\frac{-\pi}{6}$ (iv) $\frac{-\pi}{6}$ (v) $\frac{\pi}{3}$ (vi) $\frac{2\pi}{3}$

2. (i) 0 (ii) $\frac{-\pi}{3}$ (iii) $-\frac{\pi}{2}$ (iv) $\frac{\pi}{2}$

(v) π (vi) $\frac{\pi}{5}$ (vii) $\frac{-\pi}{6}$ (Viii) $\frac{\pi}{4}$

3. $\pi/5$

4. (i) $\frac{\pi}{6}$ (ii) π (iii) 1 (iv) π

5. (i) $\frac{x}{2}$ (ii) $\pi-\sec^{-1}x$ (iii) -1 (iv) $\frac{\sqrt{11}-3}{\sqrt{2}}$

6. $x+\frac{\pi}{4}$

9. 1

10. $\frac{\pi}{4}$

12. $\tan\frac{\pi}{12} = 2 - \sqrt{3}$

14. $\frac{\sqrt{5}}{3}$

16. 1

18. $\tan\frac{\pi}{12} = 2 - \sqrt{3}$

20. $\frac{\sqrt{5}}{3}$

24. **Hint:** Let $\tan^{-1} a = \alpha$

$$\tan^{-1} b = \beta$$

$$\tan^{-1} c = \gamma$$

Then given, $\alpha + \beta + \gamma = \pi$

$$\alpha + \beta = \pi - \gamma$$

Take tangent on both sides

$\tan(\alpha + \beta) = \tan(\pi - \gamma)$

26. $\emptyset = \frac{n}{n+2}$

27. $X = -1$

28. $x = -\frac{1}{2}$

29. (i) $x = -\frac{1}{12}$

(ii) $x = 0, \frac{1}{2}$

(iii) $x = 13$

(iv) $x = 1$.